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Using this fact, a little more reduction leads to the result

$$R = \frac{(a+e-f)}{24V}\sqrt{[(e-c)^2-d^2][(c-a)^2-b^2][f^2-(a-e)^2]}, \tag{3}$$

where

$$\begin{aligned}144V^2 &= a^2d^2(e^2+f^2-a^2)+b^2e^2(f^2+a^2-e^2)+c^2f^2(a^2+e^2-f^2)\\ &\quad -a^2(d^2-b^2)(d^2-c^2)-e^2(b^2-c^2)(b^2-d^2)-f^2(c^2-d^2)(c^2-b^2)-a^2e^2f^2.\end{aligned} \tag{4}$$

Seven similar expressions can be found for the radii of spheres tangent to the four edges a, c, e, f, touching one or more of the edges produced beyond the tetrahedron.

Thirdly, applying the conditions (1) to (3), we can obtain the required answer in a symmetrical form. Now (3) is the radius of a sphere touching the four edges a, c, e, f. If this sphere touches also the edges b, d, that is, if $a+d=b+e=c+f$, then

$$R = \frac{1}{24V}\sqrt[3]{\begin{aligned}&(a+b-c)(a-b+c)(-a+b+c)(b+d-f)\\&(b-d+f)(-b+d+f)(c+d-e)(c-d+e)\\&(-c+d+e)(a+e-f)(a-e+f)(-a+e+f),\end{aligned}} \tag{5}$$

where V has the value given in (4), which is already expressed in a symmetrical form. This is the required solution.

It may be added that, if A_1, A_2, A_3, A_4 are the areas of the four face-triangles, and r_1, r_2, r_3, r_4 the radii of the circles inscribed to these triangles, the above expression takes the simple form

$$R = \frac{2}{3V}\sqrt[3]{A_1A_2A_3A_4r_1r_2r_3r_4}. \tag{6}$$

If the tetrahedron is regular the expression reduces to

$$R = \frac{a}{2\sqrt{2}}.$$

Similarly the radii of the four spheres R_1, R_2, R_3, R_4 which touch three edges and the other three edges produced, are

$$R_1 = \frac{2}{3V}\sqrt[3]{A_1A_2A_3A_4r_1r_{21}r_{31}r_{41}}; \qquad R_2 = \frac{2}{3V}\sqrt[3]{A_1A_2A_3A_4r_2r_{12}r_{32}r_{42}};$$

$$R_3 = \frac{2}{3V}\sqrt[3]{A_1A_2A_3A_4r_3r_{13}r_{23}r_{43}}; \qquad R_4 = \frac{2}{3V}\sqrt[3]{A_1A_2A_3A_4r_4r_{14}r_{24}r_{34}},$$

where r_{pq} is the radius of the circle escribed to the triangle A_p which touches the edge common to A_p and A_q and the other two edges of A_p produced.

If the tetrahedron is regular, $R_1 = R_2 = R_3 = R_4 = \dfrac{3a}{2\sqrt{2}}$.

CALCULUS.

358. Proposed by C. N. SCHMALL, New York City.

About a given circle circumscribe the smallest parabola.

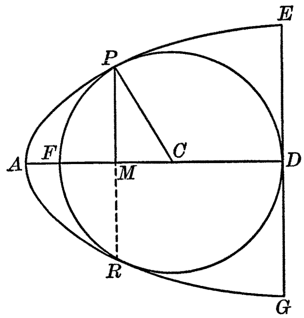
SOLUTION BY HORACE OLSON, Chicago, Illinois.

We assume that by the smallest parabola is meant the smallest segment of a parabola having its bounding ordinate tangent to the given circle.

Let $PFRD$ be the given circle whose center C is the origin of rectangular coördinates. Then, without loss of generality, the parabola may be assumed to have its axis on the axis of abscissas AD.

Let EAG be the required parabola. Then, if r is the radius of the circle, we have $x^2 + y^2 = r^2$ for the circle, and $y^2 = 2p(x + k)$ for the parabola, where p is the distance from the focus to the directrix and k is the distance AC.

Since P is the point of tangency, solving these two simultaneous equations subject to the condition of tangency, we have $r^2 - 2kp + p^2 = 0$.



Hence, $k = p/2 + r^2/2p$.
Then

$$AD = AC + CD = \frac{p}{2} + \frac{r^2}{2p} + r = \frac{(r+p)^2}{2p},$$

and

$$ED = \sqrt{2p(AD)} = (r + p).$$

Area of $EAG = a = \frac{2}{3}AD \cdot EG = \frac{4}{3}(r + p)^2/2p \cdot (r + p) = \frac{2}{3}(r + p)^3/p$.
Equating to zero the derivative of a with respect p, we have

$$\frac{da}{dp} = \frac{2}{3} \cdot \frac{3p(r+p)^2 - (r+p)^3}{p^2} = 0;$$

whence

$$(p + r)^2 = 0 \quad \text{and} \quad 2p - r = 0.$$

Hence, $p = -r$ or $p = \frac{1}{2}r$. The value $p = -r$ gives neither a maximum nor a minimum. The value $p = \frac{1}{2}r$ gives a minimum, and the equation of the corresponding parabola is $y^2 = r(x + 5r)/4$.

A similar solution was received from the PROPOSER.

359. Proposed by W. D. CAIRNS, Oberlin College.

Examine for maxima and minima

$$f(x) = e^{-cx}(1 + \cos x) \quad (c > 0)$$